

Long range triplet Josephson effect through a ferromagnetic trilayer

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(Dated: February 1, 2008)

We study the Josephson current through a ferromagnetic trilayer, both in the diffusive and clean limits. For colinear (parallel or antiparallel) magnetizations in the layers, the Josephson current is small due to short range proximity effect in superconductor/ferromagnet structures. For non colinear magnetizations, we determine the conditions for the Josephson current to be dominated by another contribution originating from long range triplet proximity effect.

PACS numbers: 74.45.+c, 74.78.Fk, 85.25.Cp.

The coexistence of superconductivity and ferromagnetism is very rare in the bulk systems. However, it can be easily achieved in the artificially fabricated superconductor/ferromagnet (S/F) heterostructures. The S/F proximity effect is characterized by the damped oscillatory behavior of the Cooper pair wave function in the ferromagnet. This phenomenon leads to the non-monotonous dependence of the critical temperature of S/F multilayers on the F layer thickness and the realization of the Josephson π -junctions (for a review see Refs. 1,2). In the diffusive limit, the proximity effect in F metal is rather short-ranged due to the large value of the ferromagnetic exchange field. This is related with the incompatibility between singlet superconductivity and ferromagnetism.

Interestingly, the non-uniform magnetization can induce the triplet superconducting correlations which are long-ranged (on the same scale as for superconductor/normal (N) metal proximity effect)³. It exists several experimental indications on this triplet proximity effect^{4,5}. However, the transition from usual to long range triplet proximity effect was never observed in the same system.

In the present work, we investigate the conditions for the observation of the Josephson current due to a long range triplet component under controllable conditions. The non-colinear magnetization may serve as a source of the long range triplet component. However, it is not possible to have the Josephson current due to the interference of the triplet and singlet components. Two sources of the triplet components are needed to observe the long range triplet Josephson effect between them. Then, the simplest experimental realization of such a situation may be the S/F'/F/F''/S system with the magnetic moments of the F', F'' layers non-colinear with the F interlayer (see Fig. 1). The optimal condition for the triplet Josephson current observation is when the thicknesses d_L and d_R of the layers F', F'' are of the order of the coherence length ξ_f in the ferromagnet. Indeed, for large d_L, d_R , the triplet component is exponentially small due to short range proximity effect in the layers F' and F'', while for very thin d_L, d_R , it is also small. Then, we predict that the magnitude of the Josephson current in the structure with F layer thickness much larger than ξ_f will be compa-

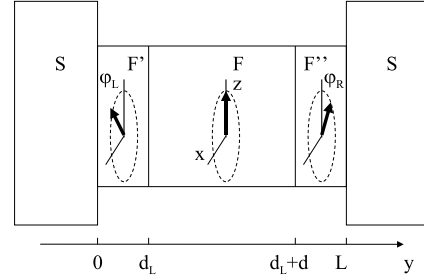


FIG. 1: Geometry of S/F'/F/F''/S junction. The arrows indicate non-colinear orientations of magnetizations in each layer with thickness d_L , d , d_R , respectively ($L = d_L + d + d_R$).

table to that of an S/N/S junction with the same length.

The similar phenomenon could be observed in lateral Josephson junctions made of a nanostructured ferromagnetic film allowing control on its magnetic domain structure. Then, the described effect would give a much larger critical current than the one predicted in S/F/S junctions with in-plane magnetic domain walls.⁶

Besides, the triplet Josephson effect provides the possibility of the 0 and π -junction realization due to the different orientations of magnetic moments in the F' and F'' layers. Such effect was revealed in $S_F/I/S_F$ junctions where S_F are magnetic superconductors with helical magnetic order separated by a thin insulating (I) layer⁷. It was also obtained in diffusive F/S multilayers with non colinear magnetizations in successive F layers⁸. In this case, the triplet Josephson effect is mediated by the inverse proximity effect in the thin S layers. It would compete with the reduction of critical temperature and gap amplitude, but these were not taken into account. In Ref. 9, an idealized circuit-theory model for the triplet proximity effect in an S/F/I/F/I/F/S junction was proposed. The spatial range of singlet and triplet proximity effect was not considered. Our work is somewhat complementary to these ones. The question about the concrete realization and optimization of the triplet Josephson effect was outside the scope of these approaches while it is of primary importance in the present study.

We also provide an analysis of the triplet Josephson current in the ballistic (clean) limit. In this case, the

singlet component reveals the non-exponential oscillatory decay but nevertheless the decay of the triplet component is even weaker and it is again possible to observe the crossover between singlet and triplet Josephson effects.

The needed conditions for the triplet proximity effect observation in the Josephson current are rather stringent. The considered system, if realized experimentally, could provide an excellent opportunity to study the crossover between the triplet and singlet Josephson effects with the rotation of the magnetic moment of any of the F layers.

Let us now calculate the supercurrent through a Josephson junction made of a ferromagnetic trilayer attached to superconducting leads, according to the geometry depicted in Fig. 1. We assume that the layers are in good electric contact and that the magnetizations in the layers have the same amplitude. The exchange field \mathbf{h} acting on the spin of the conduction electrons is parallel to the magnetizations, with following spatial dependence:

$$\mathbf{h}(y) = \begin{cases} h(\sin \phi_L \hat{x} + \cos \phi_L \hat{z}), & 0 < y < d_L, \\ h\hat{z}, & d_L < y < d_L + d, \\ h(\sin \phi_R \hat{x} + \cos \phi_R \hat{z}), & d_L + d < y < L, \end{cases} \quad (1)$$

where d_L , d , and d_R are the thicknesses of each layer, and $L = d_L + d + d_R$ is the total length of the junction. Here we adopt the same axes for space and spin quantization.

We first consider the diffusive limit, when the mean free path is shorter than the widths of the layers and coherence lengths. For simplicity, we also assume that the temperature is close to the critical temperature of the leads. Then, within the quasiclassical theory of superconductivity¹⁰, the current flowing through the junction is

$$I = \frac{GL}{e} \pi T \sum_{\omega > 0} \text{Im Tr} [\hat{F}^*(y) \hat{\sigma}_y \hat{F}'(y) \hat{\sigma}_y], \quad (2)$$

where the anomalous Green's function $\hat{F} = F_0 + \mathbf{F} \cdot \hat{\sigma}$ is a matrix in spin space and it solves the linearized Usadel equation in the ferromagnet:

$$-D\hat{F}''(y) + 2\omega\hat{F}(y) + i\mathbf{h}(y) \cdot \{\hat{\sigma}, \hat{F}(y)\} = 0 \quad (3)$$

(in units with $\hbar = k_B = 1$). Here, G is the conductance of the junction in its normal state, D is the diffusion constant of the ferromagnet, $\omega = (2n + 1)\pi T$ are the Matsubara frequencies at temperature T , $\hat{\sigma}_{i(i=x,y,z)}$ are the Pauli matrices, and the primes denote derivative along y -direction. Depairing currents generated by the orbital effect have been neglected in eq. (3), as usually done for ferromagnetic layers with in-plane magnetization¹.

The Usadel equation (3) is solved in the central F layer in terms of its values at the interfaces with F' and F''

layers:

$$\begin{aligned} F_0(y) \pm F_z(y) &= [F_0(d_L) \pm F_z(d_L)] \frac{\text{sh} q_{\pm}(d_L + d - y)}{\text{sh} q_{\pm} d} \\ &+ [F_0(d_L + d) \pm F_z(d_L + d)] \frac{\text{sh} q_{\pm}(y - d_L)}{\text{sh} q_{\pm} d}, \\ F_x(y) &= F_x(d_L) \frac{\text{sh} q_0(d_L + d - y)}{\text{sh} q_0 d} \\ &+ F_x(d_L + d) \frac{\text{sh} q_0(y - d_L)}{\text{sh} q_0 d}, \end{aligned} \quad (4)$$

and $F_y = 0$, as \mathbf{h} has no component along \hat{y} -direction. Here, $q_0 = \sqrt{2\omega/D}$ and $q_{\pm} = \sqrt{2(\omega \pm i\hbar)/D}$. As the amplitude of exchange field is much larger than critical temperature T_c , we may simplify $q_{\pm} \simeq (1 \pm i)/\xi_f$, where $\xi_f = \sqrt{D/\hbar}$ is the ferromagnet coherence length and is much shorter than superconducting coherence length $\xi_0 = \sqrt{D/2\pi T_c}$. The solutions of eq. (3) in the other layers, as well as their derivative, should match continuously eq. (4) at each interface. In absence of interface barriers with the S leads, they should also take the values $\hat{F}(y=0, L) = \hat{F}^{L,R}$, where $\hat{F}^{L,R} = (\Delta/\omega)e^{\mp i\chi/2}$ are bulk solutions in the leads. Here, Δ is the modulus of the superconducting gap and χ is the phase difference maintained between the leads. Close to T_c , the gap vanishes as $\Delta(T) = [(8\pi^2/7\zeta(3))k^2 T_c(T_c - T)]^{1/2}$. Here, we neglect selfconsistency for the gap equation in the leads, as usually done assuming that the width of S electrodes is much larger than that of F layers, or that the Fermi velocity in F layers is smaller².

To proceed further with tractable formulas, we assume that F' and F'' layers are thin: $d_L, d_R \ll \xi_f$. Then, the solution in F' layer varies only slightly with y and can be put in approximate form:

$$\begin{aligned} \hat{F}(y) &\simeq \hat{F}(d_L) + (y - d_L)\hat{F}'(d_L) \\ &- \frac{(y - d_L)^2}{d_L^2} [\hat{F}(d_L) - d_L \hat{F}'(d_L) - \hat{F}^L], \end{aligned} \quad (5)$$

which satisfies the boundary conditions at $y = 0$ and $y = d_L$. In addition, it should also solve the Usadel equation. Inserting eq. (5) into (3), we get:

$$\frac{D}{d_L^2} [\hat{F}(d_L) - d_L \hat{F}'(d_L) - \hat{F}^L] + \frac{i}{2} \mathbf{h} \cdot \{\hat{\sigma}, \hat{F}^L\} \simeq 0, \quad (6)$$

where a term $\omega \hat{F}^L$ was neglected (as $\hbar \gg T$). Equation (6) yields the results:

$$F_0(d_L) = F_0^L, \quad (7a)$$

$$F_x(d_L) = -i(d_L^2 \hbar / D) \sin \phi_L F_0^L, \quad (7b)$$

$$F_z(d_L) = -i(d_L^2 \hbar / D) \cos \phi_L F_0^L, \quad (7c)$$

provided that $d_L |\hat{F}'(d_L)| \ll |\hat{F}(d_L)|$, as can be checked consistently from eq. (4) when $d_L \ll \xi_f$.

Similar results can be obtained for $\hat{F}(y = d_L + d)$. We can now evaluate eq. (2), say at $y = d_L$, and we find

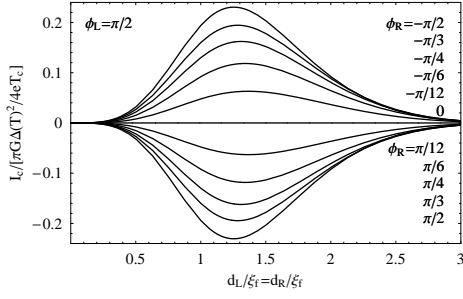


FIG. 2: Critical current induced by long range triplet proximity effect in S/F'/F/F''/S junction, in units of $(\pi G \Delta(T)^2 / 4e T_c)$, for varying length of F' and F'' layers, at $d_L = d_R \sim \xi_f \ll d \ll \xi_0$, and for different orientations of the magnetization in the layers.

$I = I_c \sin \chi$, where the critical current is:

$$I_c = \frac{2\pi T G}{e} \sum_{\omega > 0} \frac{\Delta^2}{\omega^2} \left\{ \text{Re} \frac{q_+ d}{\text{sh} q_+ d} - \frac{q_0 d}{\text{sh} q_0 d} \frac{d_L^2 d_R^2}{\xi_f^4} \sin \phi_L \sin \phi_R \right\}. \quad (8)$$

The first term in eq. (8) comes from short range singlet (F_0) and triplet (F_z) components of anomalous function \hat{F} . It equals the critical current of an S/F/S junction with length d .¹¹ Its sign oscillates with varying ratio d/ξ_f . In particular, when $d \gg \xi_f$,

$$I_{cf} = \frac{\pi G}{2\sqrt{2}e} \frac{\Delta(T)^2}{T_c} \frac{d}{\xi_f} \sin \left(\frac{\pi}{4} + \frac{d}{\xi_f} \right) e^{-d/\xi_f}. \quad (9)$$

Thus, its amplitude is also exponentially suppressed.

The second term in eq. (8) comes from long range triplet component (F_x) and yields:

$$I_{ct} = -I_{cn} (d_L^2 d_R^2 / \xi_f^4) \sin \phi_L \sin \phi_R, \quad (10)$$

where I_{cn} is the critical current in S/N/S junction:²

$$I_{cn} = \frac{G}{e} 2\pi T \sum_{\omega > 0} \frac{q_0 d}{\text{sh} q_0 d} \frac{\Delta^2}{\omega^2}. \quad (11)$$

In particular, in junctions with length $d \ll \xi_0$: $I_{cn} = (\pi G \Delta^2 / 4e T_c)$. The small prefactor $(d_L^2 d_R^2 / \xi_f^4)$ in eq. (10) comes from the simplifying assumption $d_L, d_R \ll \xi_f$ that we used in the calculation. As explained in Introduction, I_{ct} would be reduced by the exponential factor $e^{-(d_L + d_R)/\xi_f}$ at $d_L, d_R \gg \xi_f$. Thus, at optimal size $d_L, d_R \sim \xi_f$, the second term $I_{ct} \sim -I_{cn} \sin \phi_L \sin \phi_R$ is much larger than the first one, I_{cf} , provided that the magnetic layers have non colinear orientations. For arbitrary lengths $d_L, d_R \sim \xi_f$, the critical current originating from long range triplet correlation only, at $\xi_f \ll d \ll \xi_0$, was also obtained from eqs. (2) and (3), see Fig 2. We see that the triplet contribution to the critical current may be observed on the experiment only in the rather small interval of the F', F'' layers thickness: $d_L, d_R \sim (0.5 - 2.5)\xi_f$.

The dependence of the critical current with the orientations of the magnetizations in successive F layers similar to eq. (8) was obtained in Refs. 8,9. We note that

the sign of the long range component of critical current can be tuned with these orientations. This component is absent in the case of only two layers with opposite¹², or even non-colinear magnetizations¹³.

The Usadel equations would easily allow generalizing the result (8) obtained here. (i) Qualitatively, the above result should not rely on the assumption that the temperature is close to T_c and it would be preserved even at smaller temperature. (ii) Barrier interfaces between the layers and the leads would decrease both short range and long range contributions to critical current.⁹ (iii) Equation (1) may also describe the case of a ferromagnet with magnetic domains and thin domain walls (few atomic lengths). If the domain walls are large, the long range triplet contribution will be decreased by the factor $\xi_f / \delta_w \ll 1$, where δ_w is the domain wall width, in analogy with the theory of enhanced critical temperature in S/F bilayers due to domain-wall superconductivity¹⁴.

Note an interesting possibility to separate the triplet and singlet Josephson effects even for relatively thin central F layer $d \sim \xi_f$. Indeed if its thickness is around the first critical value $(3\pi/4)\xi_f$, see eq. (9), the temperature variation may serve as a fine tuning and provoke the $0/\pi$ transition^{15,16}. For the S/F'/F/F''/S system, the singlet component would vanish at such temperature and only the triplet critical current would be observed.

We consider now the clean limit. Then, the supercurrent flowing through the junction is now given by:

$$I = -\frac{2\pi T G}{e} \sum_{\omega > 0} \int \frac{d\Omega_{\mathbf{n}}}{4\pi} n_x \text{ImTr} \left[\hat{f}_{-\mathbf{n}}^*(y) \hat{\sigma}_y \hat{f}_{\mathbf{n}}(y) \hat{\sigma}_y \right], \quad (12)$$

where $\hat{f}_{\mathbf{n}}(y)$ solves the Eilenberger equation in F layer:

$$\mathbf{v} \cdot \nabla \hat{f}_{\mathbf{n}}(y) + 2\omega \hat{f}_{\mathbf{n}}(y) + i\mathbf{h} \cdot \left\{ \hat{\sigma}, \hat{f}_{\mathbf{n}}(y) \right\} = 0. \quad (13)$$

Here, $\mathbf{v} = v\mathbf{n}$ is a Fermi velocity, \mathbf{n} is a unit vector, G is the Sharvin conductance of the ballistic junction in its normal state. In addition, the solution of eq. (13) should be continuous, and match with the bulk solution in S lead where the electrons come from. That is, $\hat{f}_{\mathbf{n}}(y=0) = \hat{F}^L$ if $n_y > 0$, $\hat{f}_{\mathbf{n}}(y=L) = \hat{F}^R$ if $n_y < 0$. Again, we neglect selfconsistency for the gap equation in the leads.

Solving eq. (13) at $0 < y < d_L$ and $n_y > 0$, we find for $\hat{f}_{\mathbf{n}}(y) \equiv f_0 + \mathbf{f} \cdot \hat{\sigma}$ that:

$$\begin{aligned} f_0 \pm (\sin \phi_R f_x + \cos \phi_R f_z) &= (\Delta/\omega) e^{-i\chi/2} e^{-2(\omega \pm i\hbar)y/v_y}, \\ \sin \phi_R f_z - \cos \phi_R f_x &= 0. \end{aligned} \quad (14)$$

Then, using continuity of \hat{f} at $y = d_L$ and solving eq. (13) at $d_L < y < d_L + d$, we find:

$$\begin{aligned} f_0 \pm f_z &= \alpha e^{-2(\omega \pm i\hbar)(y-d_L)/v_y} (c_{d_L} \mp i s_{d_L} \cos \phi_L), \\ f_x &= -i\alpha \sin \phi_L s_{d_L} e^{-2\omega(y-d_L)/v_y} \end{aligned} \quad (15)$$

where $\alpha = (\Delta/\omega) e^{-i\chi/2} e^{-2\omega d_L/v_y}$, and we use short notations $s_{d_L} = \sin(2\hbar d_L/v_y)$, $c_{d_L} = \cos(2\hbar d_L/v_y)$. Similar

solution can be found for \hat{f} at $n_y < 0$. The supercurrent (12) is then conveniently evaluated at $y = d_L + d/2$ and we find $I = I_c \sin \chi$, where:

$$I_c = \frac{4\pi T G}{e} \sum_{\omega > 0} \int_0^1 dn_y n_y \frac{\Delta^2}{\omega^2} e^{-\frac{2\omega L}{v_y}} [c_d c_{d_L} c_{d_R} - c_d s_{d_L} s_{d_R} \cos \phi_L \cos \phi_R - s_d c_{d_L} s_{d_R} \cos \phi_R - s_d s_{d_L} c_{d_R} \cos \phi_L - s_{d_L} s_{d_R} \sin \phi_L \sin \phi_R]. \quad (16)$$

To proceed further, we assume that $d_L, d_R \ll \xi_f \ll d$, where the ferromagnet coherence length $\xi_f = v/h$ in clean limit is much shorter than superconducting coherence length $\xi_0 = v/2\pi T_c$. Then,

$$I_c \simeq \frac{4\pi T G}{e} \sum_{\omega > 0} \int_0^1 dn_y n_y \frac{\Delta^2}{\omega^2} e^{-\frac{2\omega d}{v_y}} \left[\cos\left(\frac{2hd}{v_y}\right) - \sin\left(\frac{2hd_L}{v_y}\right) \sin\left(\frac{2hd_R}{v_y}\right) \sin \phi_L \sin \phi_R \right]. \quad (17)$$

Here, the first term comes from short range proximity effect. It coincides with the critical current of clean S/F/S junction with length d . In particular, at $\xi_f \ll d \ll \xi_0$, it yields¹⁷:

$$I_{cf} = -\frac{\pi \Delta^2 G \xi_f}{2e T_c 2d} \sin\left(\frac{2d}{\xi_f}\right). \quad (18)$$

The second term comes from long range triplet proximity effect and yields (for $d_L \sim d_R \ll \xi_f \ll d \ll \xi_0$):

$$I_{ct} = -\frac{\pi \Delta^2 G}{2e T_c} \left[\frac{4d_L d_R}{\xi_f^2} \ln \frac{\xi_f}{2(d_L + d_R)} \right] \sin \phi_L \sin \phi_R. \quad (19)$$

It is small under assumption $d_L, d_R \ll \xi_f$. On the other hand, at $d_L, d_R \gg \xi_f$, the critical current (19) would be suppressed by the factor $\xi_f^2/d_L d_R \ll 1$, due to short range proximity effect in F' and F'' layers. Again, we expect a maximum of critical current at $d_L \sim d_R \sim \xi_f$, with amplitude $I_{ct} \propto -I_{cn} \sin \phi_L \sin \phi_R$, where $I_{cn} = (\pi \Delta^2 G / 4e T_c)$ is the critical current of a clean S/N/S junction with $d \ll \xi_0$. The dependence of the critical

current on the orientations of the magnetizations in F layers is similar to the diffusive case.

The Josephson current through a half-metal (HM) with one spin band only is expected to vanish^{5,18}. However, spin-flip processes taking place at S/F interfaces were suggested to promote triplet correlation and induce a finite supercurrent through the device^{18,19,20}. The quasiclassical theory presented in this work assumes that ferromagnetic exchange field is much smaller than the Fermi energy. Therefore, it is not well suited to address quantitatively the case of HMs, when they are comparable. Qualitatively, the non colinear layers F' and F'' with thicknesses of the atomic scale would play the role of spin flip scatterers with inverse scattering time τ_{sf}^{-1} proportional to spin band splitting h . Then, the order of magnitude for the triplet induced supercurrent can be obtained from eq. (19) by noting that the reduction factor $d_L d_R / \xi_f^2$ (up to the log term) is proportional to $1/(\tau_{sf} E_F)^2$, where E_F is Fermi energy. It is thus proportional to the probability for an electron from the minority spin band to be transferred through HM by spin-flip processes at the interfaces with the leads.

In conclusion, we determined the Josephson current through a ferromagnetic trilayer. For colinear (parallel or antiparallel) magnetizations in the layers, the Josephson current is small due to short range proximity effect in superconductor/ferromagnet structures. For non colinear magnetizations, we determined the conditions for the Josephson current to be dominated by another contribution originating from long range triplet proximity effect. In practice the triplet Josephson current may be observed in systems with the lateral layers thickness of the order of ξ_f only.

The studied structures offer an interesting possibility to study the interplay between Josephson current and dynamic precessing of the magnetic moment. Indeed we may expect the strong coupling between ferromagnetic resonance (or/and spin waves) and Josephson current - in particular the additional harmonics generation in ac Josephson effect.

We acknowledge Norman Birge and Louis Jansen for a critical reading of the manuscript and useful comments.

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